2203000205023002
EXAMINATION FEBURARY-MARCH 2024
BACHELOR OF SCIENCE (FIFTH SEMESTER)
MATHEMATICS XII
MTH-502 LINEAR ALGEBRA – I – LEVEL 2

[Time: As Per Schedule] **[Max. Marks: 50]**

Instructions:
1. Fill up strictly the following details on your answer book
- a. Name of the Examination : **BACHELOR OF SCIENCE (FIFTH SEMESTER)**
- **b.** Name of the Subject : **MATHEMATICS XII MTH-502 LINEAR ALGEBRA – I – LEVEL 2**
- c. Subject Code No : **2203000205023002**

2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. Follow usual notations.

Seat No:

Student's Signature

Q.1 Answer the following (Any Five). **10**

(1) Let $f: (x, y) \to x * y = x$
Is the binary operation * be an associative and commutative? Justify your answer.

(2) Let $L = \{(x, -x, x) \in \frac{V_3}{x} \in R\}$ then prove that L is subspace of V_3.

(3) In a vector space V, Is the set $\{0, u, v\}$ is Linearly Independent? Justify your answer.

(4) Is the sum $(XY - plane + Y - axis) = V_3$ a Direct sum? Justify your answer

(5) Find the dimension of $\left[\left(-1, {}^{3}/_{2}, 2\right), \left(3, {}^{2}/_{3}, 3\right)\right]$.

(6) In V_3; $[Y - axis\ UZ - axis] = V_3$ is true? Justify your answer

(7) Fine coordinate vector of a vector (2,3) relative to the ordered basis $\{(2,0), (0,3)\}$ of a vector space V_2.

(8) Prove that 0 is collinear with any non zero vector v in a vector space V.

Q.2 Answer the following (Any two). **10**

(1) Let R^+ be the set of positive real number. Define the operations addition and scalar multiplication as follows.
$u + v = u \cdot v$ for all $u, v \in R^+$ and $au = u^{\propto}$ for all $u \in R^+$and real scalar $\propto$.
Prove that R^+ be an abelian group with respect to addition.

(2) Let M be the set of all vectors of the form $x(4,5,6); x \in R$ in V_3. Then Prove that M is a subspace of V_3.

(3) Prove that the set $\{(x_1, x_2, x_3) \in V_3 / \sqrt{2x_1} = \sqrt{3x_2}\}$ is a subspace of V_3.

Q.3 Answer the following (Any two). **10**

(1) Let S be a non empty subset of a vector space V then prove that the span [S] is a smallest subspace of V containing S.

(2) Let $S = v_1, v_2, \ldots\ldots, v_n$ be a non empty subset of a vector space V then prove that $[v_1, v_2, v_2, \ldots\ldots\ldots, v_n] = [-1v_1, -2v_2, -3v_2, \ldots\ldots\ldots, -nv_n]$.

(3) Let S be a non empty subset of a vector space V and $u, v \in V$. If $u \in [S \cup \{v\}]$ but $u \notin [S]$ then prove that $v \in [S \cup \{u\}]$.

Q.4 Answer the following (Any two). **10**

(1) Let V be a vector space then prove that If the vectors v_1, v_2 are collinear then the set $\{v_1, v_2\}$ is L.D.

(2) In a vector space V, prove that if the set $\{v_1, v_2, \ldots\ldots\ldots\ldots, v_n\}$ is L.I. and a vector v is not a linear combination of the vectors $v_1, v_2, \ldots\ldots\ldots\ldots, v_n$ then prove that $\{v, v_1, v_2, \ldots\ldots\ldots, v_n\}$ is L.I.

(3) Determine which of the following subsets of V_3 is L.I.
i. $\{(1,5,2), (0,0,1), (1,1,0)\}$
ii. $\{(1/2, 1/3, 1), (0,0,0), (2, 3/4, -1/3)\}$

Q.5 Answer the following (Any two). **10**

(1) Verify the following results by giving one illustration to each:

i. If U and W are subspaces of finite dimensional vector space V then $\dim(U + W) = dimU + dimW - \dim(U \cap W)$.

ii. If U and W are subspaces of a finite dimensional vector spaces V such that $U \cap W = \{0\}$ then
$\dim(U \oplus W) = dimU + dimW$.

(2) If V has a basis of m elements then prove that every set of r vectors with $r > m$ is L.D.

(3) Prove that: In an n-dimensional vector space V, any set of n linearly Independent vectors is a basis.

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